PHASE NOISE CHARACTERIZATION OF SAW OSCILLATORS BASED ON A NEWTON MINIMIZATION PROCEDURE

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ABSTRACT

A Newton-Raphson iterative technique is used to optimize the values of circuit parameters which characterize a voltage-controlled SAW-stabilized oscillator (VCSO). An expression given by Parker is used to calculate double-sideband phase-noise-to-carrier ratio; good agreement between calculations and phase noise measurements is achieved by minimizing the squared-error through the use of a Newton-Raphson minimization scheme. Less accurately known circuit parameters are thus optimized in an iterative fashion using exact expressions for the elements of the Hessian matrix. This technique is useful for the accurate determination of circuit parameter values; alternatively it can be used in the design of low-phase-noise oscillators by using desired (rather than measured) phase-noise values in the objective function to be minimized.

INTRODUCTION

An understanding of the sources of phase noise in microwave oscillators is important as phase-noise specifications become increasingly stringent in state-of-the-art radar and navigation systems, for example. An expression developed by T. E. Parker can be used to predict the double-sideband, phase-noise-to-carrier ratio for a SAW-resonator-based oscillator [1,2]:

$$S_\phi(f_m) = \left[\frac{\alpha_R f_o^4}{f_m^3}\right] + \left[\frac{\alpha_E}{(2\pi\tau_g)^2 f_m^3}\right] + \left[\frac{2\alpha_R Q_L f_o^3}{f_m^2}\right] + \left[\frac{2GFKT/P_o}{(2\pi\tau_g)^2 f_m^2}\right] + \frac{\alpha_E}{f_m} + \frac{2GFKT}{P_o}, \qquad (1)$$

where f_m is the carrier offset frequency. In this equation, G and F are the compressed power gain and noise factor, respectively, of the oscillator loop amplifier; also, f_o and P_o are respectively the carrier frequency and power level at the loop amplifier output. The SAW device is characterized by the parameters τ_g, Q_L, and α_R, which are respectively the SAW group delay, loaded Q ($=\pi f_o \tau_g$) and flicker noise constant. The constant α_E is the flicker noise constant of the loop amplifier, and K and T are respectively Boltzmann's constant and the temperature in °K. In practice, α_R and α_E are determined from open-loop phase-noise measurements of the SAW device and loop amplifier.

If sufficiently accurate estimates of parameter values are available, the Parker expression can be used to calculate the phase noise characteristic of a SAW-based oscillator. In general, however, inaccuracy in the estimated values of the parameters will lead to discrepancy between the calculated values and values measured on a phase-noise measurement system.

It is possible to use a carefully-measured phase-noise characteristic to determine accurate values for less-accurately-known values of the circuit and device parameters. Our approach has been to employ a Newton-Raphson iterative technique to minimize a nonlinear function which expresses the squared error between measured phase-noise values and values computed using equation (1).

FORMULATION OF THE ALGORITHM

Consider the error function

$$\varphi(\mathbf{x}) = \sum_{i=1}^{N} [S_\phi(f_i,\mathbf{x}) - \hat{S}(f_i)]^2, \qquad (2)$$

where the first term in the brackets S_ϕ represents the calculated value of phase-noise-to-carrier ratio using equation (1) and the second term $\hat{S}$ represents the phase noise values measured at N discrete frequencies f_i, i=1,...,N. The vector **x** consists of those parameters x_k, k=1,...,M, which are to be accurately determined using the iterative minimization scheme.

In the usual fashion, we minimize the error function $\varphi(\mathbf{x})$ by locating the point in parameter space where the gradient vector of (2) vanishes, i.e. [3],

$$\mathbf{g}(\mathbf{x}) = \mathbf{0}, \qquad (3)$$

where

$$\mathbf{g}(\mathbf{x}) \equiv \left[\frac{\partial\varphi}{\partial x_1}, \ldots, \frac{\partial\varphi}{\partial x_M}\right]^T. \qquad (4)$$

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A Newton-Raphson iteration is used to drive the gradient vector to zero; an updated estimate for the parameter vector **x**, $\mathbf{x}_{v+1}$, is derived from the original parameter estimate $\mathbf{x}_v$ as

$$\mathbf{x}_{v+1} = \mathbf{x}_v - \mathbf{G}^{-1}(\mathbf{x}_v)\,\mathbf{g}(\mathbf{x}_v), \tag{5}$$

where the symmetric Hessian matrix $\mathbf{G}(\mathbf{x})$ is

$$\mathbf{G}(\mathbf{x}_v) = \left[\frac{\partial^2 \varphi}{\partial x_i\, \partial x_j}\right], \quad i,j = 1,\ldots,M. \tag{6}$$

Expressions for the gradient vector and for the elements of the Hessian matrix will be presented shortly. First, however, it is appropriate to consider the specific form of equation (2). Although the procedure for determining an optimum parameter set **x** by minimizing (2) is theoretically sound, the very high dynamic range of phase noise values encountered in practice (10 to 15 orders of magnitude) results in unacceptable numerical difficulty. For example, circuit and device effects far from the carrier frequency would be masked from "close-in" effects which would numerically dominate, due to truncation and rounding errors.

For this reason, it is expedient to modify equation (2) logarithmically as follows:

$$\tilde{\varphi}(\mathbf{x}) = \sum_{i=1}^{N} \left\{ \hat{S}_{dB}(f_i) - 10 \cdot \log_{10}\left[\frac{S_\phi(f_i,\mathbf{x})}{2}\right] \right\}^2, \tag{7}$$

where the measured phase noise values $\hat{S}_{dB}$ are now expressed in terms of decibels relative to the carrier level in a one-hertz bandwidth (dBc/Hz), and the computed values are also converted to the same units. (Our nomenclature for phase-noise-to-carrier-ratio $\hat{S}_{dB}(f)$ is also commonly denoted as $\mathcal{L}(f)$). This modification serves to ameliorate numerical difficulties. Minimizing equation (7) minimizes the squared error between the measurements (in dBc/Hz) and the calculated values; increasing the exponent from 2 to a large number would result in a minimization in the minimax sense.

Calculation of the elements of the gradient vector is straightforward. Differentiating equation (7) gives

$$\frac{\partial \tilde{\varphi}(\mathbf{x})}{\partial x_k} = \sum_{i=1}^{N} 2 \cdot \left[\hat{S}_{dB}(f_i) - 10 \cdot \log_{10}\left(\frac{S_\phi(f_i,\mathbf{x})}{2}\right)\right]\left(\frac{-10 \cdot \dfrac{\partial S_\phi}{\partial x_k}}{\ln 10 \cdot S_\phi}\right) \tag{8}$$

where the derivatives of equation (1) are evaluated for the specific parameters to be optimized. For example, if we consider the flicker noise constants α_R and α_E, then the appropriate derivatives of S_ϕ are:

$$\frac{\partial S}{\partial \alpha_R} = \frac{2Q_L f_o^3}{f_i^2} + \frac{f_o^4}{f_i^3} \tag{9}$$

and $$\frac{\partial S}{\partial \alpha_E} = \frac{1}{(2\pi\tau_g)^2 f_i^3} + \frac{1}{f_i}\,; \tag{10}$$

here the derivatives are evaluated at a specific offset frequency f_i. The elements of the Hessian matrix can also be computed explicitly:

$$\frac{\partial^2 \varphi}{\partial x_j\, \partial x_k} = \frac{-20}{\ln 10}\sum_{i=1}^{N} \frac{-10\ S'(x_j)S'(x_k)}{(\ln 10)\ S^2}$$

$$+\left(\hat{S}_{dB} - \left(\frac{10}{\ln 10}\right)\ln\frac{S}{2}\right) \cdot \frac{S\,S''(x_j x_k) - S'(x_j)S'(x_k)}{S^2}, \tag{11}$$

where S is the value computed from (1), S′ represents the first partial derivative of equation (1) with respect to the parameter in parentheses, and S″ represents the second partial derivative with respect to the parameters indicated. The function S and its derivatives are all functions of f_i; for clarity, however, the summation index has been omitted in the terms above.

The iteration proceeds by first selecting a best estimate of the parameter vector **x** from measurements of the components which comprise the voltage-controlled oscillator [2]. The appropriate derivatives of S are computed (from (9) and (10), for example) as are the elements of the gradient vector $\mathbf{g}(\mathbf{x})$ given by (8). The elements (11) of the Hessian matrix **G** are also computed and used to derive an improved estimate of the parameter vector using equation (5). The iteration then repeats until an error criterion is satisfied; in our work we have chosen to iterate until the L_2 norm of $\mathbf{g}(\mathbf{x})$ falls below a given tolerance.

Note that matrix inverse of **G** in (5) can be calculated analytically if only a few parameters are under consideration. However if **x** is comprised of many parameters, then it is best to replace (5) with

$$\mathbf{G}(\mathbf{x}_v) \cdot \Delta\mathbf{x} = -\,\mathbf{g}(\mathbf{x}_v), \tag{12}$$

where $\Delta\mathbf{x} \equiv \mathbf{x}_{v+1} - \mathbf{x}_v$. Equation (12) is solved for $\Delta\mathbf{x}$ (and thus $\mathbf{x}_{v+1}$) using a standard matrix equation solver. Because the parameters involved exhibit a wide dynamic range—F and G are on the order of magnitude of unity, while α_R is typically of the order of 10^{-38}—it is usually necessary to normalize parameters and/or employ a matrix pivoting scheme in the numerical routine which solves (12) for $\Delta\mathbf{x}$.

EXAMPLE OF THE TECHNIQUE

Figure 1 illustrates a phase-noise measurement of a 1.0 GHz voltage-controlled oscillator (VCO) stabilized using an RF Monolithics SAW device which has a typical group delay of 1 microsecond. The compressed power gain of the loop amplifier was measured to be 6.0 dB and the carrier power level at the output of the VCO loop amplifier is 16 dBm.

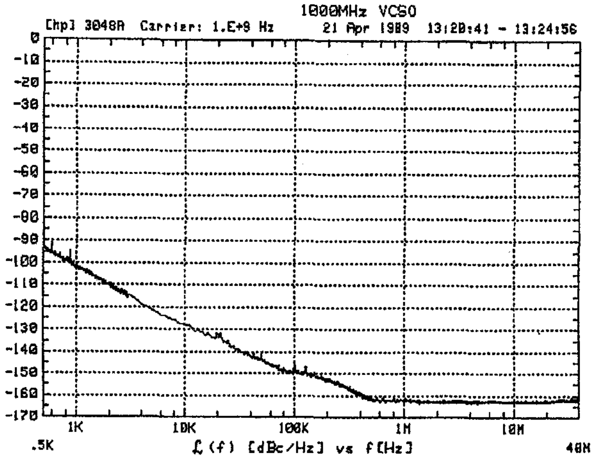

Figure 1. Phase Noise Measurement of 1GHz VCSO.

Initial estimates were made of the flicker noise constants for the SAW device and for the loop amplifier. Open-loop phase-noise measurements of those components were used to estimate α_R=9.3•10^{-38} and α_E=1.13•10^{-11}. We ignored the frequency-dependence of these parameters in our initial analysis.

Figure 2 illustrates the results of our minimization of equation (7); the summation in (7) ranged over 15 offset frequencies from 500 Hz to 400 kHz. The parameter vector **x** consisted of the parameters F, α_R, α_E, and τ_g as quantities to be varied. Initial values of 0.5µs and 50 (17 dB) were used for τ_g and F respectively; note that large values for F are not unusual for the loop amplifier, which is operating in a large-signal compressed-gain condition. Upon completion of the minimization, the parameter values were 84.8 (19.3 dB) for F, 9.54•10^{-38} for α_R, 4.02•10^{-12} for α_E, and 2.75µs for τ_g.

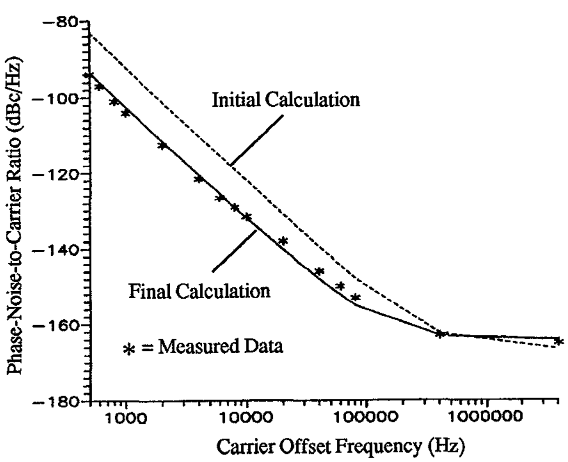

Figure 2. Calculated Phase-Noise Response Before and After Minimization

The value for τ_g is somewhat high, and in fact this quantity required numerical limits to prevent even higher values. The source of this numerical problem is likely due to the fact that the objective function (7) is very sensitive to the value of this parameter. Inaccuracy in the data for carrier offset frequencies far from the carrier results in significant changes in τ_g; investigation is continuing into this effect. This illustrates the well-established fact that a minimization approach to parameter determination requires careful attention to prevent unreasonable solutions for the parameter vector. We expect that by applying a suitable frequency-dependent weighting function in equation (7), we can reduce problems which arise from uncertainty in the data values far from the carrier.

Because the Newton-Raphson technique requires an initial parameter estimate relatively near the minimum, convergence can be somewhat slow. As a result, we found it effective to locate the minimum by initially employing the steepest-descent method for minimization (with damping). As the minimum is approached—noted by observing changes in the parameter vector at each iteration step—our algorithm switches to the Newton-Raphson technique for faster final convergence.

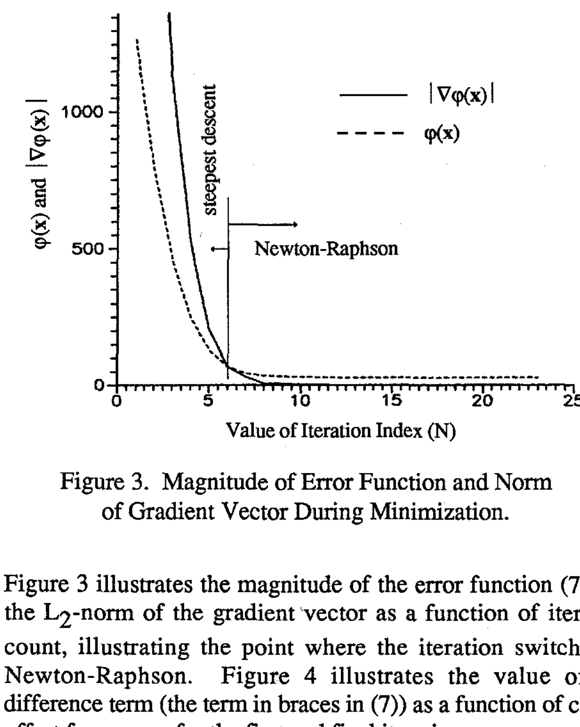

Figure 3. Magnitude of Error Function and Norm of Gradient Vector During Minimization.

Figure 3 illustrates the magnitude of the error function (7) and the L_2-norm of the gradient vector as a function of iteration count, illustrating the point where the iteration switches to Newton-Raphson. Figure 4 illustrates the value of the difference term (the term in braces in (7)) as a function of carrier offset frequency, for the first and final iterations.

CONCLUSIONS

Determination of the parameter values which relate to the phase-noise performance of a VCSO can be accomplished by minimizing a squared-error function which represents the difference between calculated and measured values of phase-noise-to-carrier ratio. An expression developed by Parker is employed in the calculation of the double-sideband phase-noise values.

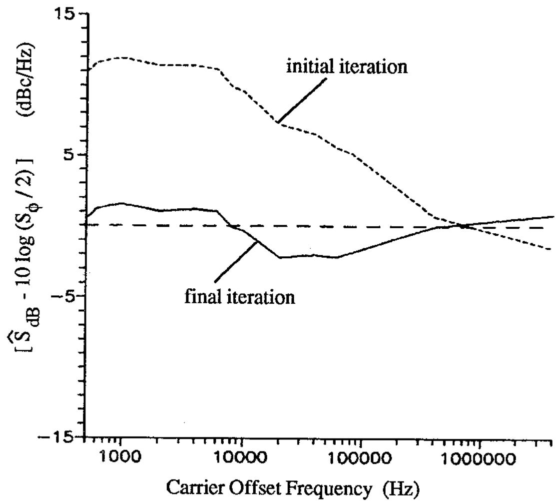

Figure 4. Discrepancy Between Calculated and Measured Phase-Noise-to-Carrier Ratio.

As in any minimization scheme, care must be employed to prevent nonphysical or meaningless solutions for the parameter vector which is to be determined. To prevent difficulty, attention should be paid to the following considerations: (i) proper choice of parameters which comprise **x** and numerical bounds which constrain **x**; (ii) specific choice of minimization scheme (steepest descent with damping versus Newton-Raphson, for instance); (iii) accurate acquisition of original measurement data; and (iv) appropriate weighting of the error function which is to be minimized. Our results suggest that with appropriate care, useful results can be obtained in the determination of VCSO circuit parameters.

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